Geometry
HS Mathematics
Unit: 12
Lesson: 01
Suggested Duration: 7 days

Exploring Polygons

Lesson Synopsis:

In this lesson, students use data collection and patterns to generate formulas for various properties of regular polygons including interior angle measures and sums, exterior angle measures and sums, and perimeter and area. Students explore formulas associated with properties of regular polygons both geometrically and algebraically using various representations.

TEKS:

G.2	***Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures.***
G.2B	**Make conjectures about angles, lines, polygons, ~~circles, and three-dimensional figures~~ and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.**
G.5	***Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems.***
G.5A	**Use numeric and geometric patterns to develop algebraic expressions representing geometric properties.**
G.5B	**Use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures ~~and solids,~~ and angle relationships in polygons ~~and circle.~~**
G.8	***Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.***
G.8A	**Find areas of regular polygons, circles, and composite figures.**
G.8F	**Use conversions between measurement systems to solve problems in real world situations.**
G.9	***Congruence and the geometry of size. The student analyzes properties and describes relationships in geometric figures.***
G.9B	**Formulate and test conjectures about the properties and attributes of polygons and their component parts based on explorations and concrete models.**

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Use patterns to develop formulas to represent properties of polygons. Make and apply generalizations about the properties of polygons. Use characteristics of polygons and regular polygons to solve problems involving area, perimeter, and angle measures given specific regular polygon(s). (G.2B; G.5A, G.5B; G.8A; G.9B)
  ELPS ELPS: 1E, 1H, 2E, 2I, 3D, 4F, 5G

Key Understandings and Guiding Questions:

- Polygons are classified by specific characteristics.
  — What are the differences between convex and concave polygons?
  — How are polygons classified?
- Formulas for geometric properties can be developed by extending patterns and making generalizations.
  — What geometric characteristics are used to develop formulas for the properties of polygons?
  — How can the different representations of geometric properties be used to connect algebra and geometry?
- Formulas for geometric properties can be used to make predictions and solve problems.
  — How can the formulas for geometric properties of polygons be used to make predictions and solve problems involving regular polygons?

Vocabulary of Instruction:

- convex
- concave
- polygons
- triangle
- quadrilateral
- pentagon
- hexagon
- heptagon
- octagon

- nonagon
- decagon
- dodecagon
- n-gon
- regular polygon
- apothem

Materials:

- graphing calculator
- chart paper
- chart markers

Resources:

- **STATE RESOURCES**
  - — **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments
    http://www.utdanacenter.org/mathtoolkit/index.php
  - — **TMT[3] Geometry:** Engage, (Polly, Polly Income Free), Explore/Explain 1 (Polygarden Landscaping Company), Student Lesson 2 (Area of Regular Polygons)
    http://www.tea.state.tx.us/math/index.html
  - — **TEXTEAMS: Geometry for All Institute:** II – Transformationally Speaking with Reflection; Act. 1 (Geometry on the Geoboard), Act. 2 (Oil Spill)

Advance Preparation:

1. Handout: **Polygons** (1 per student)
2. Handout: **Regular Polygons and Angle Relationships** (1 per student)
3. Handout: **Framing the Formulas for Polygons** (1 per student)
4. Handout: **Area of Regular Polygons** (1 per student)
5. Handout: **A Closer Look at Area of Regular Polygons** (1 per student)
6. Handout: **Applications of Regular Polygons** (1 per student)
7. Handout: **Pulling Together Polygons.** (1 per student)

Background Information:

Students rely on previous topics of study in order to carry out the explorations in this lesson. Generalization through patterns is used to develop formulas for various properties of polygons. As students develop techniques for finding the area of regular polygons, they rely on their knowledge of right triangles including Pythagorean Theorem, Pythagorean triples, special right triangles, and trigonometry.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, and substitute resources, materials, and activities to differentiate instruction to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource in your district Curriculum Developer site for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Place students in small groups.
2. Distribute copies of the handout: **Polygons** to each student. Have students work with their group to complete pp. 1-2 of the handout.
3. When groups have completed pp. 1-2 of the handout debrief in whole group instruction. Have students make corrections as needed.
   Facilitation Questions:
   - **What is a polygon? How is it defined?** *A polygon is a closed figure in a plane made up of line segments called sides that intersect only at their endpoints called vertices.*
   - **In the previous unit over quadrilaterals, the terms concave and convex were discussed. Do you think those terms apply to**

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested Day 1 (1/2 day)
MATERIALS

- chart paper (optional)
- chart markers (optional)
- Handout: **Polygons** (1 per student)

TEACHER NOTE
The purpose of the ENGAGE is to give the teacher an opportunity to pre-assess student understanding about polygons. Using the handout along with the facilitation questions is suggested but

Instructional Procedures

polygons in general? If so what do they mean? *Yes. Convex polygons are polygons such that no line containing a side of the polygon contains a point inside the interior of the polygon. When lines do contain interior points, the polygons are concave. Another way to think about convex polygons is polygons in which the diagonals of the polygon have no points on the exterior of the polygon.*

- **How many classifications by number of sides or names of polygons did you identify? What are they?** *Student answers to this question will vary. However, most students should already be familiar with some of the polygons including triangle, quadrilateral, pentagon, hexagon, octagon, etc. Most likely, students will be unfamiliar with heptagon, nonagon, decagon, or dodecagon because of unfamiliarity with the prefixes of these words.*
- **How are polygons with numerous sides, more than 12, named?** *Typically, polygons with more than 12 sides are named as n-gons, where n is the number of sides. For example, a 15-gon has 15 sides, etc.*
- **How are polygons named?** *Polygons are named by vertices either in a clockwise or counterclockwise rotation. Names of polygons must not contain a diagonal.*

4. Assign p. 3 Practice Problems to be completed individually. Have students share answers in their groups and correct as needed.

Notes for Teacher

may not be necessary. Teachers should spend only as much time in the ENGAGE phase as needed to pre-assess student vocabulary and understanding and then move to the EXPLORE phase.

STATE RESOURCES
Mathematics TEKS Toolkit: Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or used as alternate activities.

EXPLORE/EXPLAIN 1

Day 1

1. Distribute copies of the handout: **Regular Polygons and Angle Relationships** to each student.
2. Have students complete Part I (questions 1-14) with a partner or in groups.
3. This may be completed as homework if necessary.

Day 2

4. Debrief Part I in whole group discussion by asking facilitation questions. Facilitation Questions:
   - **Were you able to devise a method or determine a formula for finding the interior angle sum of a polygon?** *Yes. The interior angle sum of a convex polygon varies with the number of sides of the polygon as S = (n – 2)180, where S is the interior angle sum and n is the number of sides of the polygon. (Alternately, S = 180n – 360.)*
   - **Can you describe this relationship verbally? How does the interior angle sum change as the number of sides of the polygon change?** *In each case, as the number of sides of the polygon increases by one, the interior angle sum increases by 180.*
   - **What does this indicate about the relationship between a polygon's interior angle sum and number of sides?** *The relationship is linear.*
   - **How can you verify this (linear) relationship based on the symbolic notation?** *S = (n – 2)180 simplifies to S = 180n – 360 which is a linear equation in the form y = mx + b. The rate of change or slope is indicated by the coefficient of n, or 180.*
   - **Given the context of the situation, what is the least value that the independent value of the function can take on? Explain.** *The least value for n is 3 because a three-sided polygon or triangle is the simplest of polygons.*
   - **Given the context of the situation, what does the output values or range of this function look like? Justify excluded values.** *The output values will be positive multiples of 180 beginning with 180. Values less than 180 would indicate that a polygon has fewer than*

Suggested Days 1 – 2 (1 ½ days)
MATERIALS

- Handout: **Regular Polygons and Angle Relationships** (1 per student) continued
- graphing calculator

TEACHER NOTE
This EXPLORE phase is broken into two distinct parts. In Part I, students develop the formula for the interior angle sum of convex polygons. It is essential that this is developed and understood before posing ideas about interior and exterior angles which is what Part II deals with. In both parts, students explore the angle relationships in terms of algebra using data to generate scatterplots and develop the various formulas for the angle relationships.

TEACHER NOTE
In Algebra 1 students collect and analyze data representing inverse functions. These skills will be applied in investigating these relationships.

TEACHER NOTE
An easy way to help students realize that the function $m = \dfrac{(n-2)180}{n}$ converges to 180 is to look at an

Instructional Procedures

three sides which is not possible. Values that are not multiples of 180 would indicate polygons with number of sides that were not whole numbers which is not possible.

5. Have students complete Part II (questions 15-21) with a partner or in small groups. This can be completed for homework, if necessary.
1. Debrief Part II in whole group discussion. This may be completed at the beginning of the next period.
   Facilitation Questions:
   - **What procedure did you use in order to find the measure of one interior angle of a given regular polygon once you knew the interior angle sum?** *Divide the interior angle sum by the number of sides which is also the number of interior angles.*
   - **Since you discovered that the interior angle sum for any convex polygon is given by *S = (n – 2)180 or S = 180n – 360*, how could you express the measure of an interior angle of any convex polygon?** $\frac{(n-2)180}{n}$
   - **What relationship did you discover between an interior angle of a polygon and its exterior angle? Explain.** *They are supplementary because they form a linear pair.*
   - **How could you express the measure of an exterior angle of any convex polygon based on the relationship you just described?** $180-\frac{(n-2)180}{n}$
   - **In each case, what did you discover about the exterior angle sum of convex polygons?** *The exterior angle sum of any convex polygon is 360°.*
   - **Since the exterior angle sum of any convex polygon is always 360°, what is an alternative to $180-\frac{(n-2)180}{n}$ that could be used to find the measure of an exterior angle of a regular convex polygon? Explain.** *Since the exterior angle sum is always 360° and the polygon is regular, dividing 360 by the number of sides (which is also the number of exterior angles; one at each vertex) will give the measure of the exterior angle.*
   - **According to the data in your table, what did you discover about the measure of the interior angle of a regular polygon as the number of sides increased?** *The measure of the interior angle increased as the number of sides increased.*
   - **How would you describe the relationship between the measure of the interior angle and the number of sides of the regular polygon? Is it linear? Why or why not?** *The relationship is not linear. The amount of change in the measure of the interior angle is not constant as the number of sides vary by one each time. Alternately, the symbolic relationship* $\frac{(n-2)180}{n}$ *is not linear because of division by n.*
   - **Given the context of the situation, what is the maximum value that the function $m=\frac{(n-2)180}{n}$ can take on? Explain.** *The function takes on values closer and closer to 180, but never exactly 180. Letting the function equal 180 and solving for n yields a false statement, which means the function can never be exactly equal to 180. Geometrically, if the function takes on a value of 180, each interior angle would have a value of 180 or a straight angle and the*

Notes for Teacher

equivalent form. $m=\frac{(n-2)180}{n}$ can also be written as $m=\frac{(n-2)}{n}180$.
Have students examine the behavior of $\frac{(n-2)}{n}$ as *n* gets extremely large; the ratio converges to 1 which means the function converges to 180.

Instructional Procedures

figure would not "close."

- **Given the context of the situation and the relationship between the interior and exterior angles of a polygon, what can you conclude about the extreme values of an exterior angle of a regular polygon?** Recall that the interior and exterior angles of a polygon at a vertex are supplementary. *Since the measure of the interior angle approaches 180°, the measure of the exterior angle approaches 0°. Since the least interior angle measure is 60° (the regular triangle), the greatest exterior angle measure is 120°.*

Notes for Teacher

ELABORATE 1

1. Distribute copies of the handout: **Framing the Formulas for Polygons** to students. Verify that the students' discoveries agree with the formulas. Clarify any misconceptions.
2. Have students complete the Practice Problems. This may be completed as homework, if necessary.

Suggested Day 3
MATERIALS

- Handout: **Framing the Formulas for Polygons** (1 per student)

TEACHER NOTE
In this activity students will apply the formulas developed for polygons to determine missing measure in various situations involving polygons.

STATE RESOURCES
TEXTEAMS: Geometry for All Institute: II – Transformationally Speaking with Reflection; Act. 1 (Geometry on the Geoboard) may be used to reinforce these concepts or used as alternate activities.

EXPLORE 2

1. Distribute the handout: **Area of Regular Polygons** to each student.
2. Have students complete problems #1-2 with a partner or in groups.
3. When groups have finished debrief results in whole group discussion.
4. Assign problems #3-4 to be completed independently. This can be completed as homework, if necessary.

Suggested Day 4
MATERIALS

- Handout: **Area of Regular Polygons** (1 per student)

TEACHER NOTE
In this activity, students will investigate various methods for finding the area of regular polygons.

TEACHER NOTE
If students need additional exploration of the area formula for regular polygons, this activity can be extended to investigate a regular hexagon using the same techniques.

EXPLAIN 2

1. In whole group discussion have volunteer students share their summaries on problems #3-4 from **Area of Regular Polygons**.
2. Debrief with facilitation questions.
   - **What was the result of finding the center of the square and drawing a radius to each vertex?** *The radii divide the square into four non-overlapping congruent triangular regions.*
   - **Suppose the radii of any regular polygon are drawn. Do you think the polygon would be divided into non-overlapping congruent**

Suggested Day 5
MATERIALS

- Handout: **A Closer Look at Area of Regular Polygons** (1 per student)
- graphing calculator

TEACHER NOTE
In this activity students will be introduced to the formula for area of

Instructional Procedures

triangular regions? Why or why not? *Yes. Since the radii are congruent and the figure's side lengths are congruent (the figure is regular), the non-overlapping triangles are congruent by SSS.*

- **Once the square was divided into non-overlapping triangular regions, describe the technique used to find its area.** *Since the four non-overlapping triangular regions are congruent, the total area of the square is found by finding the area of one of the triangles and multiplying by four.*
- **Since drawing the radii in any regular polygon results in non-overlapping congruent triangular regions, do you think a similar technique could be used to find the area of other regular polygons? Explain.** *Yes. The area of any regular polygon can be found by finding the area of one of the triangular regions and multiplying by the number of triangular regions (the number of sides of the polygon).*
- **Can you write a formula for finding the area of the square in terms of the apothem and the side length of the square? If so, what is it?** *Yes. For the square, the formula would be* $A_{square} = 4[\frac{1}{2}(s)(a)]$ *where s is the side length of the square and a is the apothem.*
- **In the formula** $A_{square} = 4[\frac{1}{2}(s)(a)]$**, what is the significance of the 4?** *The 4 represents the four triangular regions.*
- **Since multiplication is commutative and associative, suppose we rearrange the formula so that it becomes** $A_{square} = \frac{1}{2}(4s)(a)$**. What is the significance of the term *4s*?** *It is the perimeter of the square.*
- **Since *4s* represents the perimeter of the square, what do you suppose the formula for the area of any regular polygon would be?** $A_{regularpoly} = \frac{1}{2}(a)(p)$ *where a is the length of the apothem and p is the perimeter of the regular polygon.*

3. Distribute the handout: **A Closer Look at Area of Regular Polygons** to each student.
4. Review the notes on the first page of handout: **A Closer Look at Area of Regular Polygons** and clarify any misconceptions.
5. Demonstrate example problems as needed.
6. Have students complete the Practice Problems. This handout can be completed as homework, if necessary.

Notes for Teacher

regular polygons ($A = \frac{1}{2}ap$) and apply the formula to solve problems.

TEACHER NOTE
The answer to the second facilitation question may not be obvious to students as they only had the opportunity to explore the technique using a square. The teacher may wish to demonstrate that the triangular regions are indeed congruent by facilitating a discussion with another regular polygon. Since the formula or technique for finding the area of regular polygons is developed using a square, students most likely will not see the value in the process. The formula $A_{square}=s^2$ is much easier to use! The square was chosen because the reference triangle used in the process is convenient for them as it is a 45°-45°-90°. Teachers may need to point out that although the technique works for the square, and even the regular triangle, it is intended to be used for regular polygons for which there are no convenient area formulas such as polygons with 5 or greater sides.

MISCONCEPTION
Since the process for developing the formula for area of a regular polygon, $A = \frac{1}{2}ap$, is developed using a square, and the reference triangle is a 45°-45°-90°, some students may over generalize that the reference triangle for every regular polygon will be a 45°-45°-90°. Students must use available tools to solve right triangles: Pythagorean Theorem, Pythagorean triples, special right triangles, and trig ratios.

ELABORATE 2

1. Distribute the handout: **Applications of Regular Polygons** to each student.
2. Have students work in pairs to complete the problems.
3. If time allows, put students in groups of four and assign each group one of the problems to display on chart paper. Have groups post results and lead the class in a whole group discussion of each problem.

Suggested Day 6

MATERIALS

- Handout: **Applications of Regular Polygons** (1 per student)
- graphing calculator
- chart paper
- chart markers

TEACHER NOTE
In this activity students have the opportunity to use and extend their knowledge of area of regular polygons.

Instructional Procedures

Notes for Teacher

Students find partial areas of regular polygons and revisit the effect of scale factor on area.

STATE RESOURCES
TMT^3 Geometry: Engage, (Polly, Polly Income Free), Explore/Explain 1 (Polygarden Landscaping Company), Student Lesson 2 (Area of Regular Polygons) may be used to reinforce these concepts or used as alternate activities.

EVALUATE

1. Distribute the handout: **Pulling Together Polygons** to each student.
2. Have students work independently to complete the problems to assess their understanding of the characteristics and applications of polygons.

Suggested Day 7

MATERIALS

- Handout: **Pulling Together Polygons.** (1 per student)
- graphing calculator

TEACHER NOTE
Students will work independently on this activity. This activity should be used to assess student understanding of concepts covered in this lesson.

STATE RESOURCES
TEXTEAMS: Geometry for All Institute: II – Transformationally Speaking with Reflection; Act. 2 (Oil Spill) may be used as an introductory activity for the next unit on circles.

TAKS CONNECTION

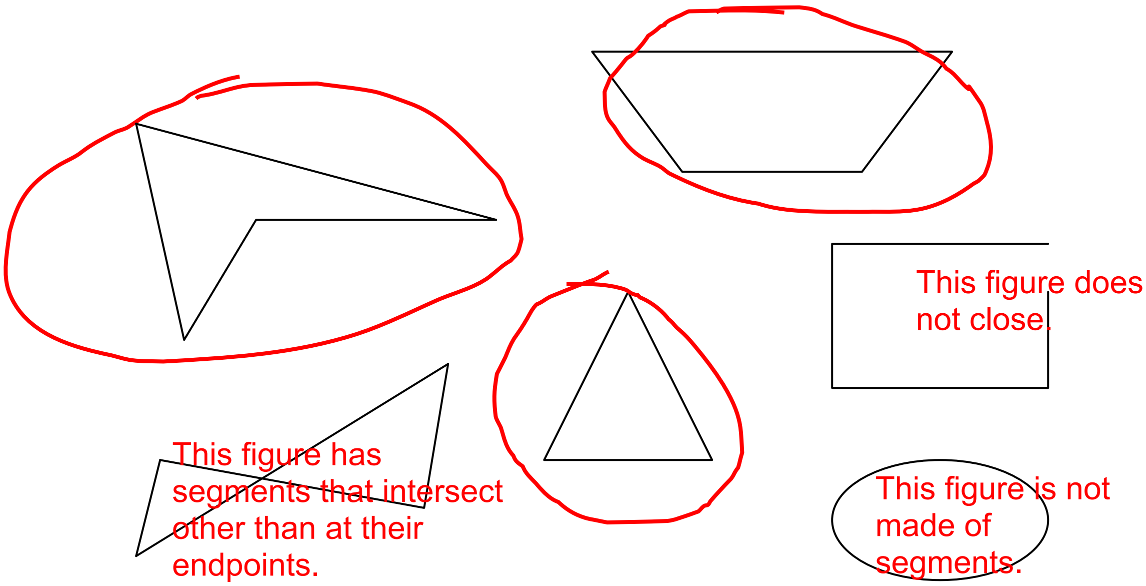
Grade 11 TAKS 2003 #34

Grade 11 TAKS 2004 #5

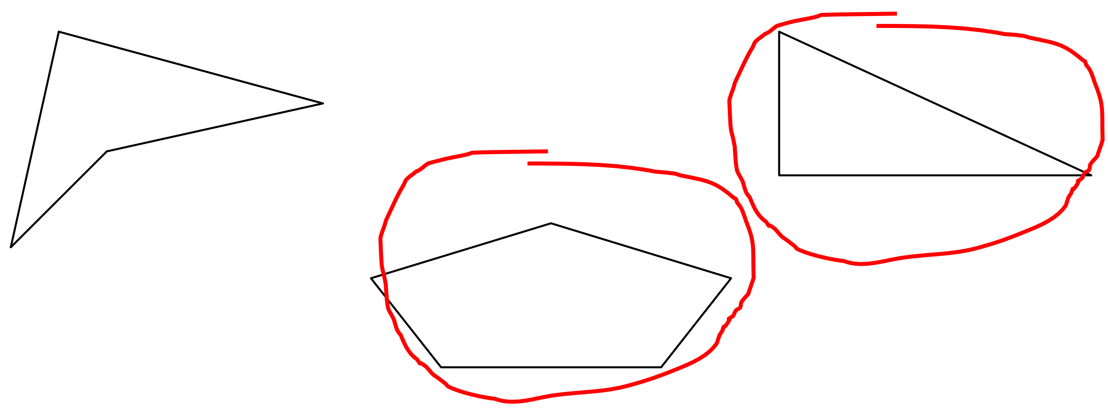
Grade 11 TAKS 2006 #38
Grade 11 July TAKS 2006 #6,18,54

Polygons (pp. 1 of 3) KEY

Circle the figures that are polygons. If the figure is not a polygon, give a justification.



Determine if the polygons below are convex or concave. Circle the convex polygons.



Polygons (pp. 2 of 3) KEY

Match the name of the polygon with its representative figure.

E Pentagon

F Decagon

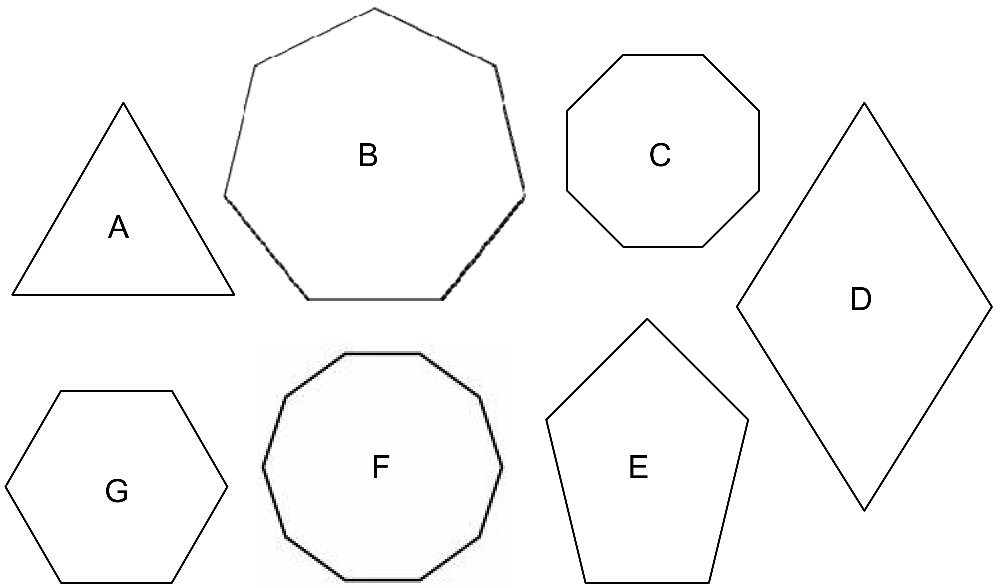
A Triangle

C Octagon

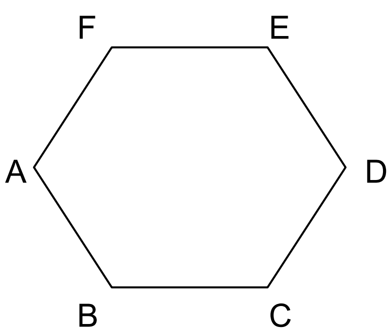
B Heptagon

G Hexagon

D Quadrilateral



Is there more than one way to name the polygon below? Explain the procedure for naming polygons. Give an example and non-example.



Yes there is more than one way to name it.

Polygons are named by vertices either in a clockwise or counterclockwise rotation.

Examples:
Hexagon ABCDEF or Hexagon EDCBAF

Non-Examples:
Hexagon ABDCEF or FCEDAB

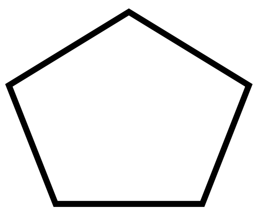
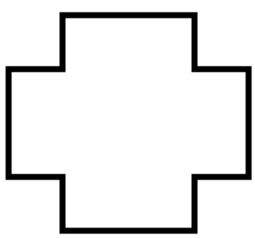
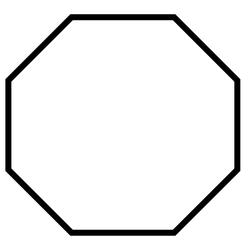
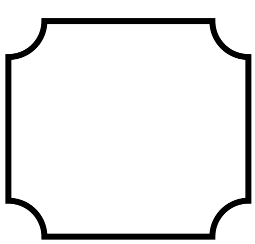
Give a congruence statement that would have to be true if the figure above was a regular hexagon.

Example:
$AB \cong CD$ or any side combination
$\angle B \cong \angle C$ or any angle combination

Polygons (pp. 3 of 3) KEY

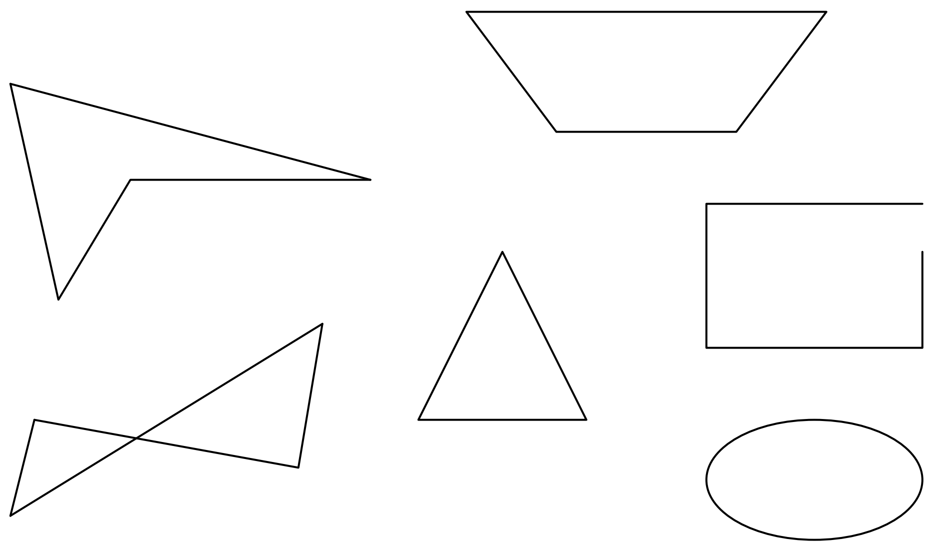
Practice Problems

For each figure, identify it as a polygon or non-polygon. If it is a polygon, identify it as convex or concave and classify it according to number of sides.

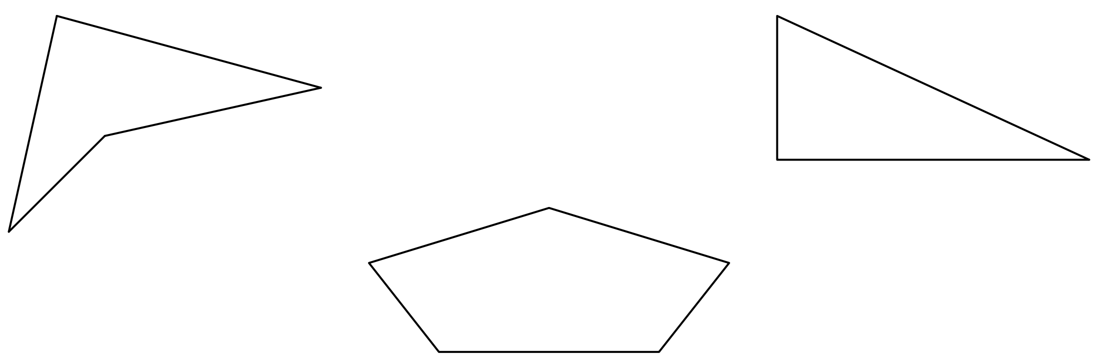
Figure	Polygon or Non-polygon	Convex or Concave	Classification of Polygon
	Polygon	convex	quadrilateral
	Polygon	convex	pentagon
	Polygon	concave	dodecagon
	Polygon	convex	octagon
	Non-Polygon	n/a	n/a

Polygons **(pp. 1 of 3)**

Circle the figures that are polygons. If the figure is not a polygon, give a justification.



Determine if the polygons below are convex or concave. Circle the convex polygons.



Polygons **(pp. 2 of 3)**

Match the name of the polygon with its representative figure.

___________ Pentagon

___________ Decagon

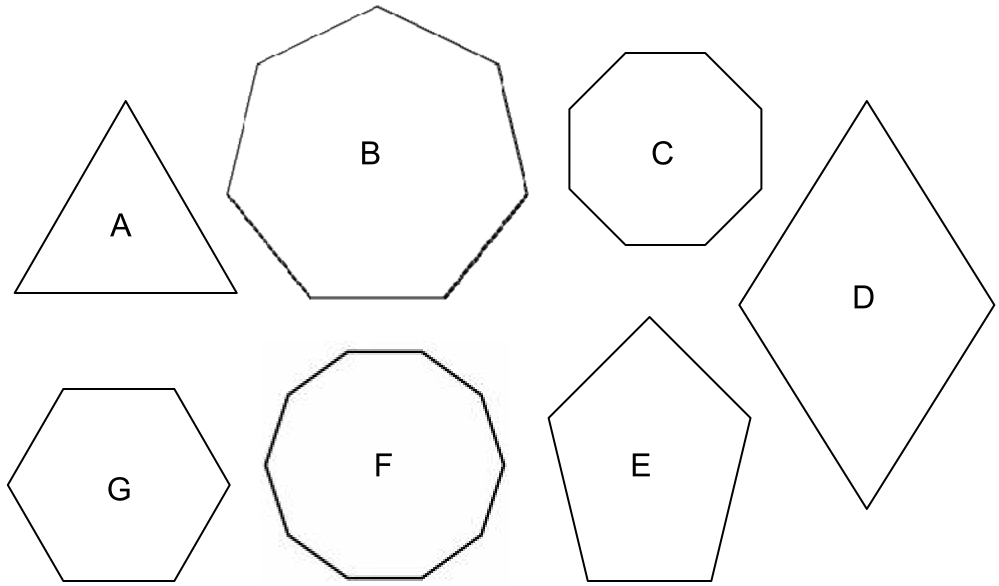
___________ Triangle

___________ Octagon

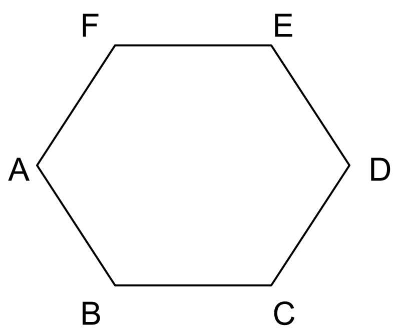
___________ Heptagon

___________ Hexagon

___________ Quadrilateral



Is there more than one way to name the polygon below? Explain the procedure for naming polygons. Give an example and non-example.

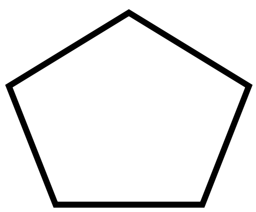
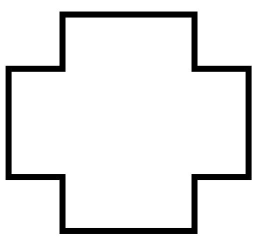
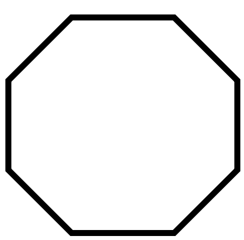
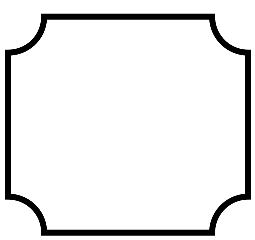


Give a congruence statement that would have to be true if the figure above was a regular hexagon.

Polygons (pp. 3 of 3)

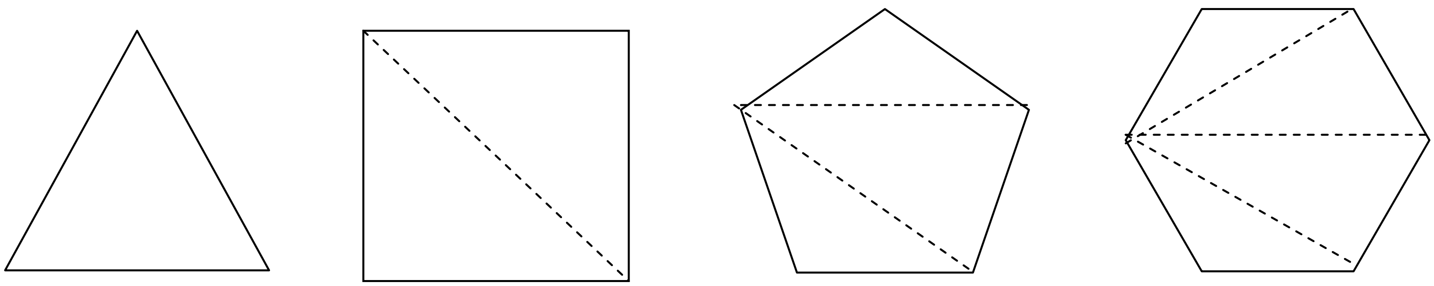
Practice Problems

For each figure, identify it as a polygon or non-polygon. If it is a polygon, identify it as convex or concave and classify it according to number of sides.

Figure	Polygon or Non-polygon	Convex or Concave	Classification of Polygon
			
			
			
			
			

Regular Polygons and Angle Relationships (pp. 1 of 7) KEY

Part I
In Unit 3, Geometric Patterns, conjectures were made about the interior angle sum for polygons. A *regular* polygon is both equilateral and equiangular. Suppose each of the polygons below is *regular* and divided into triangular regions as shown.



1. What is the interior angle sum for the triangle? How do you know?
   180°. Triangle Angle Sum Theorem.

2. Based on your answer to question 1, what is the interior angle sum of the regular quadrilateral? Explain.
   360°. Since there are two triangular regions, the interior angle sum is 180°+180° or 360°.

3. Based on your answer to question 1, what is the interior angle sum of the regular pentagon? Explain.
   540°. Since there are three triangular regions, the interior angle sum is 180°+180°+180°

4. Use your reasoning to complete the table below.

Polygon Name	Number of Sides	Number of Triangular Regions	Process to Find the Sum of the Interior Angles	Sum of the Interior Angles
Triangle	**3**	1	(1)180	**180**
Quadrilateral	4	2	(2)180	360
Pentagon	5	3	(3)180	540
Hexagon	6	4	(4)180	720
***n*-gon**	n	n - 2	(n – 2)180	180n - 360

5. Based on the data in the table, describe how the angle sum changes as additional sides are added to the polygon?
   Each time an additional side is added to the polygon, the interior angle sum increases by 180.

6. What does your answer to question 5 lead you to believe about the algebraic relationship between the interior angle sum and the number of sides of the polygon?
   Since the rate of change of the interior angle sum is constant, the relationship between the interior angle sum and the number of sides of the polygon is linear.

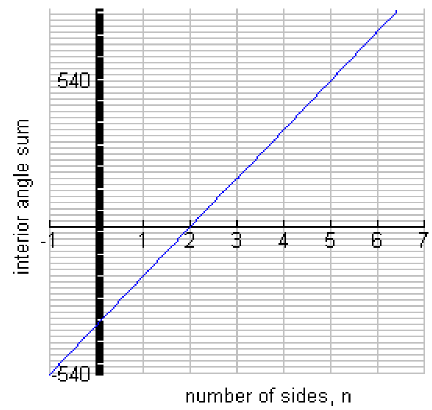
Regular Polygons and Angle Relationships (pp. 2 of 7) KEY

7. Based on the data in your table, write a function for the interior angle sum of a polygon in terms of the number of sides of a polygon.
   s = (*n* - 2)180 or *s* = 180*n* – 360

8. What is the significance of the coefficient of the variable that represents the number of sides of the polygon in your function? Explain.
   It is the rate of change of the function or the slope of the graph of the function. The interior angle sum increases by 180 each time an additional side is added to the polygon.

9. Sketch a graph of the function in question 7.

Number of Sides (*n*)	Interior Angle Sum
-1	-540
0	-360
1	-180
2	0
3	180
4	360
5	540



10. What is the domain and range of the function you graphed in question 9?
    The domain and range are both *all real numbers.*

11. Are there parts of the graph that do not represent the pattern generated by the data in the table? Explain. What is an appropriate domain and range given the context of the problem setting? Explain your reasoning.
    Yes. Number of sides less than three is meaningless since the simplest polygon is a three-sided figure. The interior angle sums will always be a positive multiple of 180, since adding an additional side results in increasing the angle sum by 180.
    Domain = {counting numbers greater than or equal to three}
    Range = {positive multiples of 180}

12. Use the function you wrote in question 7 to predict the interior angle sum for a 12 sided polygon.
    1800

13. Use the function you wrote in question 7 to determine the type of polygon that has an interior angle sum of 1080.
    8 sided or octagon

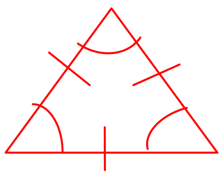
14. Is it possible to have a polygon with an interior angle sum of 2430? Why or why not?
    No. 2430 is not divisible by 180, which means 2430 is not an element of the range given the context of the problem. Alternately, solving for the number of sides that would result in an interior angle sum of 2430 means the polygon would have 15.5 sides, which is not possible.

Regular Polygons and Angle Relationships (pp. 3 of 7) KEY

Part II

In the previous activity, you discovered the relationship between the interior angle sum of a regular polygon and the number of sides of the regular polygon. Recall that a *regular* polygon is both *equiangular* and *equilateral*.

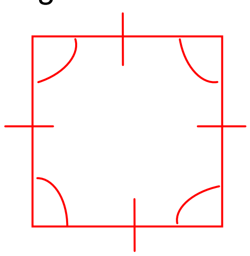
15. Sketch a *regular* triangle below. What is the interior angle sum for the triangle?



The interior angle sum is 180.

   a. Write and solve an equation to find the measure of each of the angles of the regular triangle.
      $x + x + x = 180$; therefore, $x = 60$. Each interior angle has a measure of 60°.
   b. Suppose you extend one side of the triangle so that an exterior angle is formed. What is measure of the exterior angle formed? How do you know?
      The measure of the exterior angle is 120° since it forms a linear pair with the interior angle.
   c. Suppose you construct one exterior angle at each vertex. What is the exterior angle sum for the triangle? Explain your reasoning.
      Since the triangle is regular, each of the exterior angles is 120°, and since there are three exterior angles, the exterior angle sum is 360°.

16. Sketch a *regular* quadrilateral below. What type of quadrilateral did you sketch? What is the interior angle sum for the quadrilateral?



Square. The interior angle sum is 360.

   a. Write and solve an equation to find the measure of each of the angles of the regular quadrilateral.
      $x + x + x + x = 360$; therefore, $x = 90$. Each interior angle has a measure of 90°.
   b. Suppose you extend one side of the quadrilateral so that an exterior angle is formed. What is measure of the exterior angle formed? How do you know?
      The measure of the exterior angle is 90° since it forms a linear pair with the interior angle.
   c. Suppose you construct one exterior angle at each vertex. What is the exterior angle sum for the quadrilateral? Explain your reasoning.
      Since the quadrilateral is regular, each of the exterior angles is 90°, and since there are four exterior angles, the exterior angle sum is 360°.

Regular Polygons and Angle Relationships (pp. 4 of 7) KEY

17. Repeat the procedure to find the measure of each of the interior and exterior angles of a regular pentagon, regular hexagon, regular heptagon, and regular octagon as well as the exterior angle sum. Record your data in the table below.

Polygon Name	Number of Sides, *n*	Sum of the Interior Angles $(n - 2)180$	Process to Find the Measure of an Interior Angle	Measure of Each Interior Angle	Measure of Each Exterior Angle	Exterior Angle Sum (one angle at each vertex)
Triangle	3	180	180/3	60°	120°	360°
Quadrilateral	4	360	360/4	90°	90°	360°
Pentagon	5	540	540/5	108°	72°	360°
Hexagon	6	720	720/6	120°	60°	360°
Heptagon	7	900	900/7	128.57°	51.43°	360°
Octagon	8	1080	1080/8	135°	45°	360°
***n*-gon**	n	$180n - 360$	$\frac{(n-2)180}{n}$	$\frac{(n-2)180}{n}$	$180 - \frac{(n-2)180}{n}$	360°

a. Describe the process for finding the measure of an interior angle of a regular polygon.
Divide the interior angle sum of the polygon by the number of sides of the polygon (which is also the number of interior angles of the polygon).

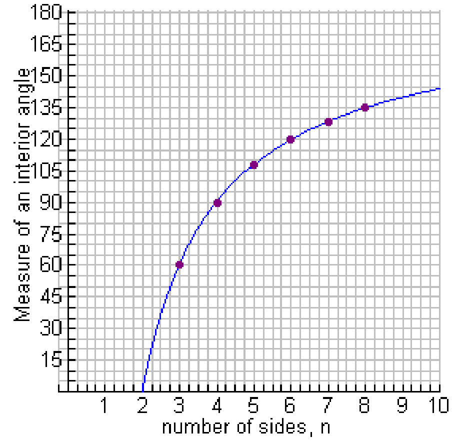
b. Describe the process for finding the measure of an exterior angle of a regular polygon.
Since the interior and exterior angles form a linear pair, they are supplementary. Therefore, the measure of the exterior angle can be found by subtracting the measure of the interior angle from 180.

c. Based on the data in the table, describe how the measure of an interior angle changes as the number of sides of the regular polygon changes. What does this lead you to believe about the relationship between the measure of an interior angle of a polygon and the number of sides of a regular polygon?
The measure of an interior angle of a regular polygon increases as the number sides increases but not by the same amount each time; therefore, the relationship is not linear.

Regular Polygons and Angle Relationships (pp. 5 of 7) KEY

18. Use your graphing calculator and the list editor to analyze the data from your table.
    a. Enter the number of sides of the polygon into L_1 and the measure of an interior angle into L_2. Create a scatterplot of your data using the STAT PLOT menu of your graphing calculator. Sketch the graph below.

Number of Sides (*n*)	Measure of Interior Angle
3	60
4	90
5	108
6	120
7	128.57
8	135



    b. Enter the expression for the measure of an interior angle of a regular *n*-gon from your table as Y_1 and graph it along with the scatterplot. Sketch the graph along with your scatterplot in 18.a.
    c. Generate a table of values for Y_1 using your graphing calculator. Start your table at zero and increment the table by 1. How do the values in the table compare to the values in your data table?
       Some of the (x, y) values for Y_1 are the same as those in the data table. Some values such as (0, ERR), (1, -180), and (2, 0) are not part of the data table.
    d. Based on your answer to 18.c and given the context of the measure of an interior angle of a regular polygon as a function of the number of sides of the regular polygon, what is an appropriate domain and range? Explain your reasoning.
       The independent variable takes on values that represent the number of sides of a regular polygon, therefore, domain = {counting numbers greater than or equal to three}, and range = {y such that $y = [(x – 2)180/x]$ and x is an element of the domain}.
    e. Use the function from your table and that you graphed as Y_1 in the calculator to find the following values. What is the measure of an interior angle of a regular 750-gon? Regular 1000-gon? Regular 10000-gon? Regular 10500-gon?
       179.52°, 179.64°, 179.964°, 179.966°
    f. Based on your answers in 18.e, what is the suggested maximum value of the function?
       180°
    g. Suppose the function takes on a value of 180°, find the input value (number of sides of the polygon). Explain the meaning of your answer both algebraically and geometrically.
       Algebraically: $180 = \frac{(n-2)180}{n}$ Attempts to solve for *n* yields a false statement such as 0 = -360, 0 = -2, etc. This is obviously a false statement which means the function can never take on a value of 180°
       Geometrically: If each of the interior angles measured 180°, the figure would not "close" because the interior angles would be straight angles. Similarly, the exterior angles would each have measure 0°; therefore, the exterior angle sum would never equal 360°.

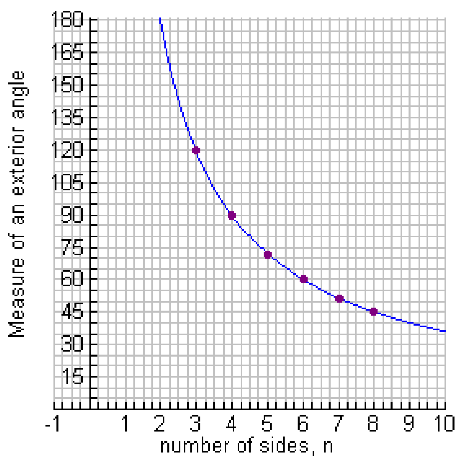
Regular Polygons and Angle Relationships (pp. 6 of 7) KEY

h. Describe the measure of an interior angle of a regular polygon as a function of the number of sides of the polygon verbally.
The measure of an interior angle of a regular polygon begins at 60° and increases as the number of sides of the polygon increases according to the rule $\frac{(n-2)180}{n}$ and approaches the value 180°.

i. Based on your previous answers and your knowledge of the relationship between the interior angle of a polygon and its exterior angle, describe the measure of an exterior angle of a regular polygon as a function of the number of sides of the regular polygon. Explain your reasoning.
An interior angle of a regular polygon and its exterior angle are supplementary. Since the measure of an interior angle of a regular polygon begins at 60° and increases as the number of sides of the polygon increases according to the rule $\frac{(n-2)180}{n}$ and approaches the value 180°, its supplement, the measure of an exterior angle begins at 120° and decreases as it approaches 0°

j. Create a scatterplot of the measure of an exterior angle of a regular polygon versus the number of sides of the regular polygon using the graphing calculator. Sketch the graph below.

Number of Sides (n)	Measure of Exterior Angle
3	120°
4	90°
5	72°
6	60°
7	51.43°
8	45°



k. Enter the expression for the measure of an exterior angle of a regular n-gon from your table as Y_2 and graph it along with the scatterplot. Sketch the graph along with your scatterplot in 18.j.

l. Given the context of the situation, describe an appropriate domain and range for the function.
The independent variable takes on values that represent the number of sides of a regular polygon, therefore, domain = {counting numbers greater than or equal to three}, and range = {y such that $y = 180 - [(x - 2)180/x]$ and x is an element of the domain}.

Regular Polygons and Angle Relationships (pp. 7 of 7) KEY

19. Suppose a regular polygon has an interior angle that measures 150°. Write an equation and find the number of sides of the polygon. What is the measure of one of its exterior angles? Explain your reasoning.

    The number of sides can be found by the equation $150 = \frac{(n-2)180}{n}$. Solving the equation gives n = 12. The polygon has 12 sides. Since the interior angle has a measure of 150°, the exterior angle is the supplement or 30°. Similarly, since the exterior angle sum is always 360°, the number of sides can be found using the value of the exterior angle and the equation $\frac{360}{30} = n$.

20. Suppose a regular polygon has an exterior angle that measures 20°. Find the measure of one of its interior angles and the number of sides of the polygon. Explain your reasoning.
    The measure of the interior angle is the supplement to 20° or 160°. The number of sides of the polygon can be found by $\frac{360}{20} = n$ or $160 = \frac{(n-2)180}{n}$ therefore, n = 18.

21. Is it possible for a regular polygon to have an interior angle that measures 175°? Why or why not?

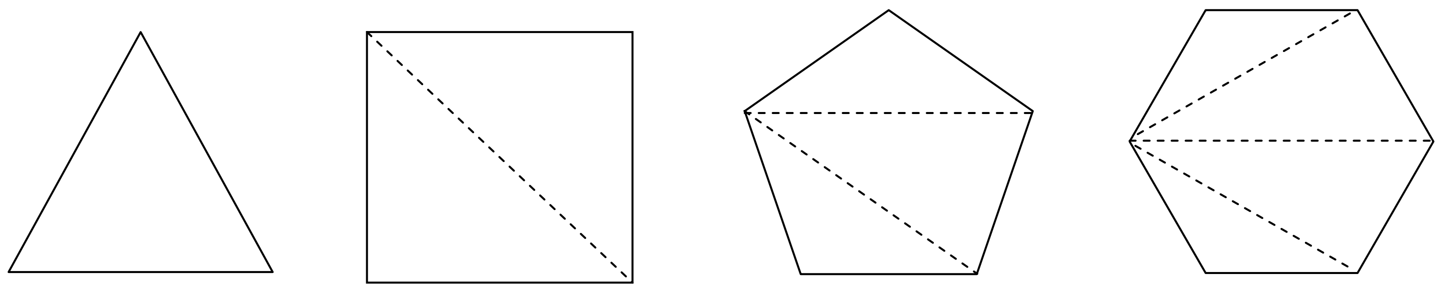
    Yes. Solving the equation $175 = \frac{(n-2)180}{n}$ gives n = 72, so the polygon would have 72 sides.

    Similarly, the measure of the exterior angle is the supplement to 175° or 5°. Solving $\frac{360}{5} = n$ gives n = 72.

Regular Polygons and Angle Relationships (pp. 1 of 7)

Part I

In Unit 3, Geometric Patterns, conjectures were made about the interior angle sum for polygons. A *regular* polygon is both equilateral and equiangular. Suppose each of the polygons below is *regular* and divided into triangular regions as shown.



1. What is the interior angle sum for the triangle? How do you know?

2. Based on your answer to question 1, what is the interior angle sum of the regular quadrilateral? Explain.

3. Based on your answer to question 1, what is the interior angle sum of the regular pentagon? Explain.

4. Use your reasoning to complete the table below.

Polygon Name	Number of Sides	Number of Triangular Regions	Process to Find the Sum of the Interior Angles	Sum of the Interior Angles
Triangle	**3**			**180**
Quadrilateral				
Pentagon				
Hexagon				
***n*-gon**				

5. Based on the data in the table, describe how the angle sum changes as additional sides are added to the polygon?

6. What does your answer to question 5 lead you to believe about the algebraic relationship between the interior angle sum and the number of sides of the polygon?

Regular Polygons and Angle Relationships (pp. 2 of 7)

7. Based on the data in your table, write a function for the interior angle sum of a polygon in terms of the number of sides of a polygon.

8. What is the significance of the coefficient of the variable that represents the number of sides of the polygon in your function? Explain.

9. Sketch a graph of the function in question 7.

10. What is the domain and range of the function you graphed in question 9?

11. Are there parts of the graph that do not represent the pattern generated by the data in the table? Explain. What is an appropriate domain and range given the context of the problem setting? Explain your reasoning.

12. Use the function you wrote in question 7 to predict the interior angle sum for a 12 sided polygon.

13. Use the function you wrote in question 7 to determine the type of polygon that has an interior angle sum of 1080.

14. Is it possible to have a polygon with an interior angle sum of 2430? Why or why not?

Regular Polygons and Angle Relationships (pp. 3 of 7)

Part II

In the previous activity, you discovered the relationship between the interior angle sum of a regular polygon and the number of sides of the regular polygon. Recall that a *regular* polygon is both *equiangular* and *equilateral*.

15. Sketch a *regular* triangle below. What is the interior angle sum for the triangle?

    a. Write and solve an equation to find the measure of each of the angles of the regular triangle.

    b. Suppose you extend one side of the triangle so that an exterior angle is formed. What is measure of the exterior angle formed? How do you know?

    c. Suppose you construct one exterior angle at each vertex. What is the exterior angle sum for the triangle? Explain your reasoning.

16. Sketch a *regular* quadrilateral below. What type of quadrilateral did you sketch? What is the interior angle sum for the quadrilateral?

    a. Write and solve an equation to find the measure of each of the angles of the regular quadrilateral.

    b. Suppose you extend one side of the quadrilateral so that an exterior angle is formed. What is measure of the exterior angle formed? How do you know?

    c. Suppose you construct one exterior angle at each vertex. What is the exterior angle sum for the quadrilateral? Explain your reasoning.

Regular Polygons and Angle Relationships (pp. 4 of 7)

17. Repeat the procedure to find the measure of each of the interior and exterior angles of a regular pentagon, regular hexagon, regular heptagon, and regular octagon as well as the exterior angle sum. Record your data in the table below.

Polygon Name	Number of Sides, *n*	Sum of the Interior Angles (n – 2)180	Process to Find the Measure of an Interior Angle	Measure of Each Interior Angle	Measure of Each Exterior Angle	Exterior Angle Sum (one angle at each vertex)
Triangle	3	180				
Quadrilateral						
Pentagon						
Hexagon						
Heptagon						
Octagon						
***n*-gon**						

a. Describe the process for finding the measure of an interior angle of a regular polygon.

b. Describe the process for finding the measure of an exterior angle of a regular polygon.

c. Based on the data in the table, describe how the measure of an interior angle changes as the number of sides of the regular polygon changes. What does this lead you to believe about the relationship between the measure of an interior angle of a polygon and the number of sides of a regular polygon?

Regular Polygons and Angle Relationships (pp. 5 of 7)

18. Use your graphing calculator and the list editor to analyze the data from your table.
    a. Enter the number of sides of the polygon into L_1 and the measure of an interior angle into L_2. Create a scatterplot of your data using the STAT PLOT menu of your graphing calculator. Sketch the graph below.

    b. Enter the expression for the measure of an interior angle of a regular *n*-gon from your table as Y_1 and graph it along with the scatterplot. Sketch the graph along with your scatterplot in 18.a.

    c. Generate a table of values for Y_1 using your graphing calculator. Start your table at zero and increment the table by 1. How do the values in the table compare to the values in your data table?

    d. Based on your answer to 18.c and given the context of the measure of an interior angle of a regular polygon as a function of the number of sides of the regular polygon, what is an appropriate domain and range? Explain your reasoning.

    e. Use the function from your table and that you graphed as Y_1 in the calculator to find the following values. What is the measure of an interior angle of a regular 750-gon? Regular 1000-gon? Regular 10000-gon? Regular 10500-gon?

    f. Based on your answers in 18.e, what is the suggested maximum value of the function?

    g. Suppose the function takes on a value of 180°, find the input value (number of sides of the polygon). Explain the meaning of your answer both algebraically and geometrically.

Regular Polygons and Angle Relationships (pp. 6 of 7)

h. Describe the measure of an interior angle of a regular polygon as a function of the number of sides of the polygon verbally.

i. Based on your previous answers and your knowledge of the relationship between the interior angle of a polygon and its exterior angle, describe the measure of an exterior angle of a regular polygon as a function of the number of sides of the regular polygon. Explain your reasoning.

j. Create a scatterplot of the measure of an exterior angle of a regular polygon versus the number of sides of the regular polygon using the graphing calculator. Sketch the graph below.

k. Enter the expression for the measure of an exterior angle of a regular *n*-gon from your table as Y_2 and graph it along with the scatterplot. Sketch the graph along with your scatterplot in 18.j.

l. Given the context of the situation, describe an appropriate domain and range for the function.

Regular Polygons and Angle Relationships (pp. 7 of 7)

19. Suppose a regular polygon has an interior angle that measures 150°. Write an equation and find the number of sides of the polygon. What is the measure of one of its exterior angles? Explain your reasoning.

20. Suppose a regular polygon has an exterior angle that measures 20°. Find the measure of one of its interior angles and the number of sides of the polygon. Explain your reasoning.

21. Is it possible for a regular polygon to have an interior angle that measures 175°? Why or why not?

Framing the Formulas for Polygons Key

Several formulas can be used to calculate measures of angles and number of diagonals in polygons. In the formulas, *n* represents the number of sides.

- In convex polygons, the sum of the interior angles is $(n-2)180^\circ$.
- Measure of each interior angle of a regular polygon is $\frac{(n-2)180^\circ}{n}$.
- In convex polygons, the sum of the exterior angles is 360°.
- Measure of each exterior angle of a regular polygon is $\frac{360^\circ}{n}$.

Practice Problems

1. What is the interior angle sum of a hexagon?
   720°
2. What is the measure of an exterior angle of a regular heptagon?
   ≈51.4°
3. What is the measure of an interior angle of a regular decagon?
   144°
4. If a regular polygon has an interior angle sum of 1980°, how many sides does the polygon have?
   13
5. If the measure of an exterior angle of a regular polygon is 45°, how many sides does the polygon have? What is the measure of the interior angle?
   The polygon has 8 sides. The interior angle has a measure of 135°.

Framing the Formulas for Polygons

Several formulas can be used to calculate measures of angles and number of diagonals in polygons. In the formulas, *n* represents the number of sides.

- In convex polygons, the sum of the interior angles is ____________________________.

- Measure of each interior angle of a regular polygon is ____________________________.

- In convex polygons, the sum of the exterior angles is ____________________________.

- Measure of each exterior angle of a regular polygon is ____________________________.

Practice Problems

1. What is the interior angle sum of a hexagon?

2. What is the measure of an exterior angle of a regular heptagon?

3. What is the measure of an interior angle of a regular decagon?

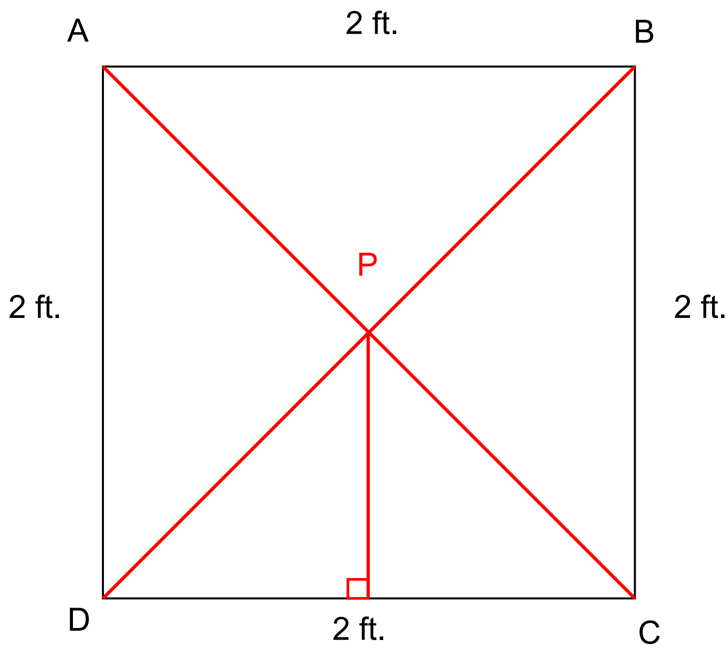
4. If a regular polygon has an interior angle sum of 1980°, how many sides does the polygon have?

5. If the measure of an exterior angle of a regular polygon is 45°, how many sides does the polygon have? What is the measure of the interior angle?

Area of Regular Polygons (pp. 1 of 2) KEY

Let's consider two different ways to find the area of a regular quadrilateral or square. Use Figure 1 for the following questions.

Figure 1



Method 1:

1. Suppose Figure 1 is a square with side length 2 ft. Use the formula for the area of a square to find the area of Figure 1. Show your work below.
   Area of a square is the length of its side, squared or $A_{SQUARE} = s^2$; therefore, $A_{SQUARE} = (2 \text{ ft.})^2$ or $A_{SQUARE} = 4 \text{ ft.}^2$

Method 2:

2. Locate the center of the square, the point equidistant from the vertices, and label this point P. Draw, $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ in the figure.
   a. Each of these segments is called a *radius* for the regular quadrilateral. What do you know to be true about each of these segments?
      Since P is equidistant from the vertices of the square, $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ are congruent to each other.
   b. $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ divide the square into four triangular regions. What do you suppose is true about each of the four triangles formed? Justify your reasoning.
      They are congruent to each other by SSS. Since the quadrilateral is regular, each triangle has a side length that is 2 ft. Therefore, one pair of congruent sides is identified between the triangles. Since P is equidistant from the vertices of the square, $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ are congruent to each other; therefore, the triangles have three pairs of congruent sides and SSS.

Area of Regular Polygons (pp. 2 of 2) KEY

c. Draw the altitude of ΔPDC from vertex P. Label the altitude $\overline{PX}$. When referring to regular polygons, this segment is called the *apothem*; a segment that extends from the center of a polygon perpendicularly to opposite side. What do you think is true about ΔDPX and ΔCPX? Explain your reasoning.

Since $\overline{PD} \cong \overline{PC}$ and $\overline{PX} \cong \overline{PX}$, $\Delta DPX \cong \Delta CPX$ by HL.

d. Based on your answer to 2.c, what can you conclude about $\overline{DX}$ and $\overline{CX}$? Explain your reasoning.

$\overline{DX} \cong \overline{CX}$ by CPCTC. It follows that DX = CX = 1 ft.

e. What can you conclude about $\angle PDC$? Explain your reasoning.

Since $\angle ADP \cong \angle PDC$ by CPCTC and $m\angle ADP + m\angle PDC = 90$ by Angle Addition Postulate, it follows that $m\angle PDC = 45$.

f. Based on your answers to the previous questions, find the length of the altitude $\overline{PX}$. Explain your reasoning.

ΔPDX is a 45°-45°-90° triangle since DX = 1 ft., PX = 1 ft.

g. Find the area of ΔPDC. How does the area of ΔPDC compare to the area of the square? How can this information be used to find the area of the square?

The area of ΔPDC is $\frac{1}{2}(2ft.)(1ft.)$ or 1 ft.^{2}. The area of ΔPDC is ¼ the area of the square; therefore, the area of the square can be found by finding the area of ΔPDC and multiplying by four.

h. Based on your answer to 2. g, find the area of the square using the area of ΔPDC. Show your work below.

$A_{SQUARE}=4(A_{TrianglePDC})$ or $4[\frac{1}{2}(2ft.)(1ft.)]$ or 4 ft.^{2}

i. How does the area as calculated using Method 2 compare to the area you calculated using Method 1?

They are the same.

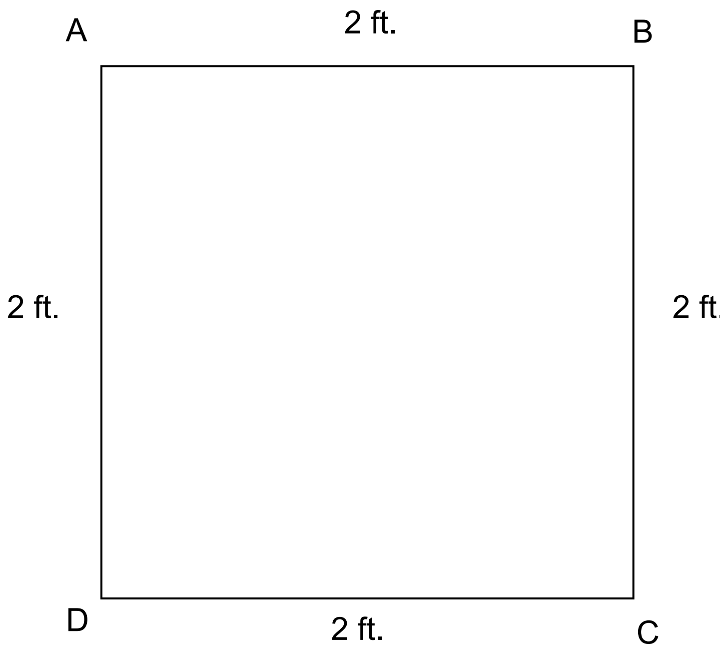
3. Summarize Method 2 as used to find the area of the regular quadrilateral or square.
To find the area of the regular quadrilateral, the square is divided into four non-overlapping, congruent, triangular regions. The area of the square is found by finding the area of one of the triangular regions and multiplying by 4.

4. Do you think Method 2 could be used to find the areas of other regular polygons? Why or why not?
Yes, any regular polygon can be divided into non-overlapping, congruent, triangular regions. The area of the regular polygon could be found by finding the area of one of the triangular regions and multiplying by the number of sides of the polygon.

Area of Regular Polygons **(pp. 1 of 2)**

Let's consider two different ways to find the area of a regular quadrilateral or square. Use Figure 1 for the following questions.

Figure 1



Method 1:

1. Suppose Figure 1 is a square with side length 2 ft. Use the formula for the area of a square to find the area of Figure 1. Show your work below.

Method 2:

2. Locate the center of the square, the point equidistant from the vertices, and label this point P. Draw, $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ in the figure.
   a. Each of these segments is called a *radius* for the regular quadrilateral. What do you know to be true about each of these segments?

   b. $\overline{PA}$, $\overline{PB}$, $\overline{PC}$, and $\overline{PD}$ divide the square into four triangular regions. What do you suppose is true about each of the four triangles formed? Justify your reasoning.

Area of Regular Polygons (pp. 2 of 2)

c. Draw the altitude of $\triangle PDC$ from vertex P. Label the altitude $\overline{PX}$. When referring to regular polygons, this segment is called the *apothem*; a segment that extends from the center of a polygon perpendicularly to opposite side. What do you think is true about $\triangle DPX$ and $\triangle CPX$? Explain your reasoning.

d. Based on your answer to 2.c, what can you conclude about $\overline{DX}$ and $\overline{CX}$? Explain your reasoning.

e. What can you conclude about $\angle PDC$? Explain your reasoning.

f. Based on your answers to the previous questions, find the length of the altitude $\overline{PX}$. Explain your reasoning.

g. Find the area of $\triangle PDC$. How does the area of $\triangle PDC$ compare to the area of the square? How can this information be used to find the area of the square?

h. Based on your answer to 2.g, find the area of the square using the area of $\triangle PDC$. Show your work below.

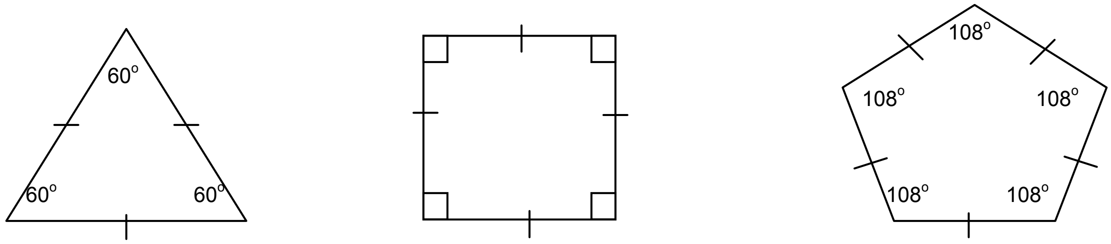
i. How does the area, as calculated using Method 2, compare to the area you calculated using Method 1?

3. Summarize Method 2 as used to find the area of the regular quadrilateral or square.

4. Do you think Method 2 could be used to find the areas of other regular polygons? Why or why not?

A Closer Look at Area of Regular Polygons (pp. 1 of 3) KEY

A regular polygon is a convex polygon in which all sides are congruent and all angles are congruent.



Equilateral triangle | Square | Regular pentagon

Any regular polygon can be divided into non-overlapping, congruent, triangular regions. The area of the regular polygon can be found by finding the area of one of the triangular regions and multiplying by the number of sides of the polygon. This method can be shown to reduce to the formula…

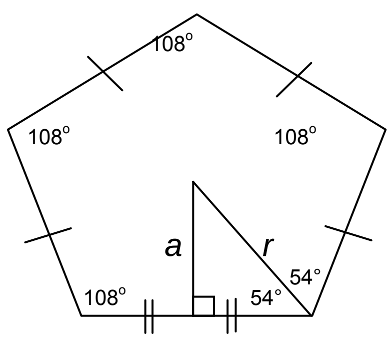
Area of a regular polygon = $A = \frac{1}{2}ap$

where *p* represents the perimeter and *a* represents the *apothem*.

Note: The addition of non-overlapping regions to find the area of a regular polygon is possible because of the Area Addition Postulate which states that the area of a figure is the sum of the areas of its non-overlapping regions.

The apothem is a segment that extends from the center of a regular polygon perpendicularly to a side of the regular polygon. The apothem is also the perpendicular distance from the center of the figure to a side. The apothem bisects a side of the regular polygon. In Figure 1 below, *a* is the length of the apothem.

The radius is a segment that extends from the center of the regular polygon to a vertex of the polygon. The radius is also the distance from the center of the polygon to a vertex of the polygon. The radius bisects an interior angle of the regular polygon. In Figure 1 below, *r* is the length of the radius.



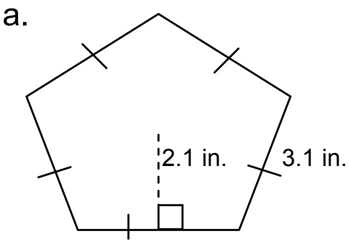
Attributes of **special right triangles**, **45°-45°-90°,** and **30°-60°-90°,** and the **trigonometric ratios** can be used to solve problems involving regular polygons.

A Closer Look at Area of Regular Polygons (pp. 2 of 3) KEY

EXAMPLES

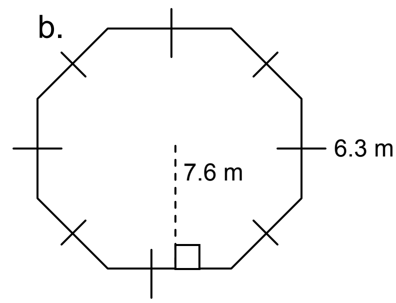
1. Find the perimeter and areas of the following regular polygons.

a.



P=15.5 in.
A=16.275 in.^{2}

b.



P=50.4 in.
A=191.52 in.^{2}

2. A regular hexagon has an area of 18 square centimeters. If the apothem measures 2.4 centimeters, find the length of each side.
Length of each side, s=2.5 cm.

3. An equilateral triangle has a side length of 14 meters and an apothem of 4.04. If the dimensions are doubled, how does this affect the area? Why?

$A_{original}$=84.84 m^2
A_{double}=339.36 m^2

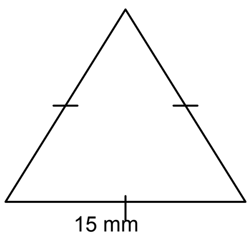
A_{double}: $A_{original}$ is a ratio of 4:1. Therefore, doubling the dimensions increases the area by a factor of 4.

A Closer Look at Area of Regular Polygons (pp. 3 of 3) KEY

Practice Problems

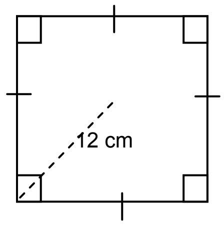
Find the apothem for each of the regular polygons below. Then find the perimeter and area for each figure.

1.



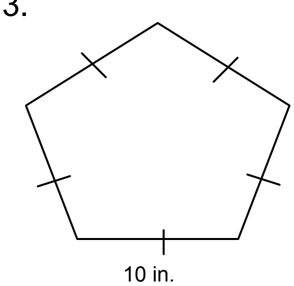
a ≈ 4.33 mm
P = 45 mm
A ≈ 97.4 mm^2

2.



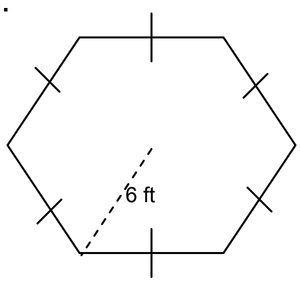
a ≈ 8.5 cm
P ≈ 68 cm
A ≈ 288 cm^2

3.



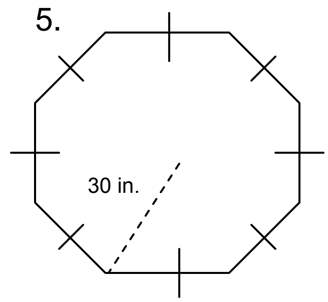
a ≈ 6.88 in.
P = 50 in.
A ≈ 172 in.^{2}

4.



a = $3\sqrt{3}$ ft or ≈ (5.2 ft)
P = 36 ft.
A ≈ 93.5 ft.^{2}

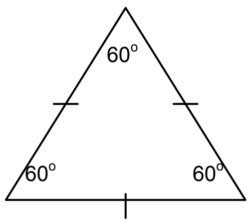
5.



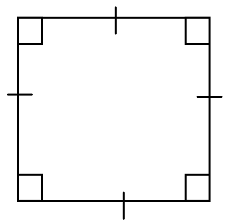
a ≈ 27.7 in.
P ≈ 184 in.
A ≈ 2548.4 in.^{2}

A Closer Look at Area of Regular Polygons (pp. 1 of 3)

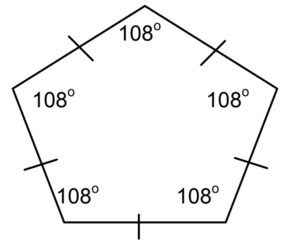
A ____________________ polygon is a convex polygon in which all sides are congruent and all angles are congruent.



Equilateral triangle



Square



Regular pentagon

Any regular polygon can be divided into non-overlapping, congruent, triangular regions. The area of the regular polygon can be found by finding the area of one of the triangular regions and multiplying by the number of sides of the polygon. This method can be shown to reduce to the formula…

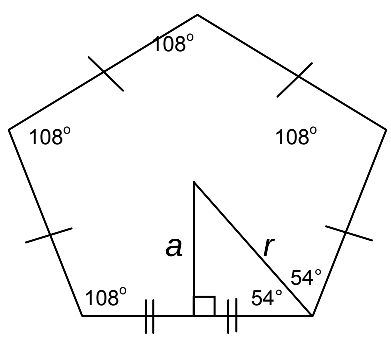
Area of a regular polygon = ____________________________

where *p* represents the perimeter and *a* represents the *apothem*.

Note: The addition of non-overlapping regions to find the area of a regular polygon is possible because of the _______________________________ which states that the area of a figure is the sum of the areas of its non-overlapping regions.

The ___________________is a segment that extends from the center of a regular polygon perpendicularly to a side of the regular polygon. The _________________ is also the perpendicular distance from the center of the figure to a side. The _____________________ bisects a side of the regular polygon. In Figure 1 below, *a* is the length of the apothem.

The _______________________ is a segment that extends from the center of the regular polygon to a vertex of the polygon. The _____________________ is also the distance from the center of the polygon to a vertex of the polygon. The _________________ bisects an interior angle of the regular polygon. In Figure 1 below, *r* is the length of the radius.



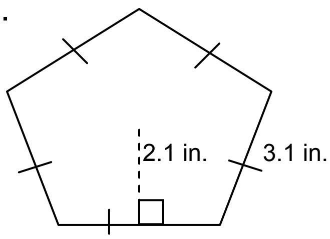
Attributes of **special right triangles**, **45°-45°-90°,** and **30°-60°-90°,** and the **trigonometric ratios** can be used to solve problems involving regular polygons.

A Closer Look at Area of Regular Polygons (pp. 2 of 3)

EXAMPLES

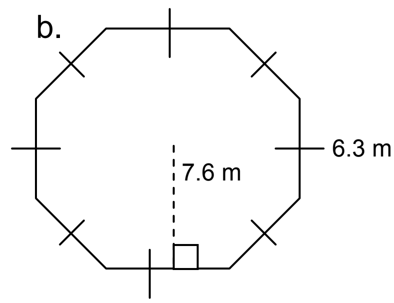
4. Find the perimeter and areas of the following regular polygons.

a.



P = ___________
A = ___________

b.



P = ___________
A = ___________

5. A regular hexagon has an area of 18 square centimeters. If the apothem measures 2.4 centimeters, find the length of each side.

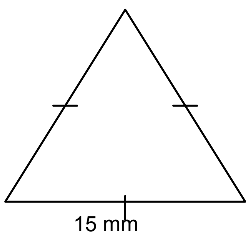
6. An equilateral triangle has a side length of 14 meters and an apothem of 4.04. If the dimensions are doubled, how does this affect the area? Why?

A Closer Look at Area of Regular Polygons (pp. 3 of 3)

Practice Problems

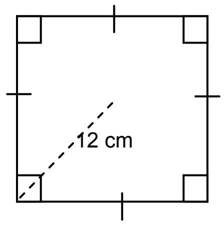
Find the apothem for each of the regular polygons below. Then find the perimeter and area for each figure.

1.



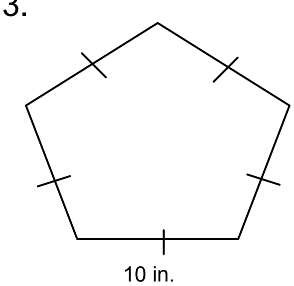
a ≈ ___________
P = ___________
A ≈ ___________

2.



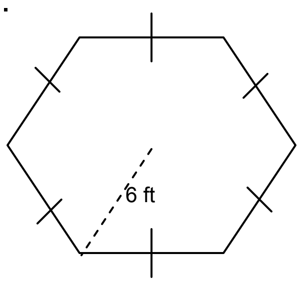
a ≈ ___________
P ≈ ___________
A ≈ ___________

3.



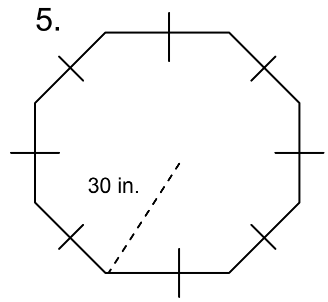
a ≈ ___________
P = ___________
A ≈ ___________

4.



a = ___________
P = ___________
A ≈ ___________

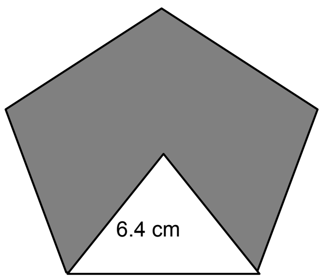
5.



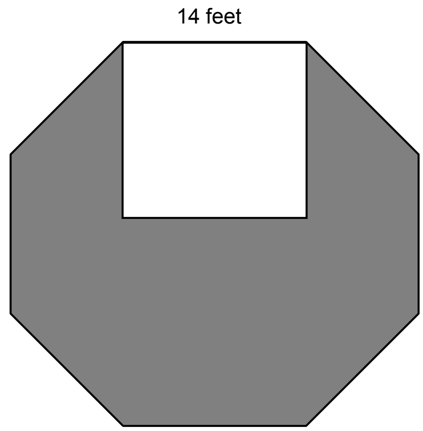
a ≈ ___________
P ≈ ___________
A ≈ ___________

Applications of Regular Polygons (pp. 1 of 2) KEY

1. Find the area of the shaded regions. Assume that both the pentagon and octagon are regular. Round to the nearest tenth.



$A_{shaded} \approx 77.9\ cm^2$



$A_{shaded} \approx 750.4\ ft.^2$

2. Mary is tiling the entry to her new house in white tiles. In the center of the white tiles, she wants to place blue tiles in the form of a regular hexagon. The perimeter of the hexagon will be 216 inches. How many square **meters** of area will the hexagon occupy? (1 inch = 2.54 centimeter, 1 meter = 100 centimeters)
$A \approx 2.2\ m^2$

Applications of Regular Polygons (pp. 2 of 2) KEY

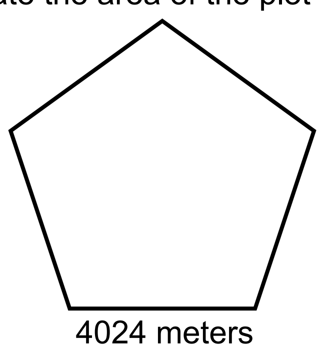
3. Bill is building a garden in the shape of a regular octagon that has an area of 482.8 square yards. If the apothem measures approximately 12 yards, what is the perimeter of the garden? How many **feet** of fencing will Bill need to enclose the garden?
   P≈80.5 yds. or approximately 241.5 ft. of fence.

4. The best selling pool advertised by Waterman's Pools and Supply is a regular pentagon inscribed in a circle. The portion of the pool outside the pentagon is tiled for walking and sitting.
   a. If the radius of the circle is 10 feet, what would be the surface area of the pool itself?
      `A≈238.95 ft.^{2}

   b. If Sue wanted to have the pool built in her backyard but wanted the dimensions increased one and one-half times the original size, what would be the surface area of her pool?
      A≈538 ft.^{2}

   c. Assuming the pool's dimensions are increased one and one-half times the original size, by what factor will the area increase?
      The area increases by a scale factor of 9:4 or 2.25 times the original area.

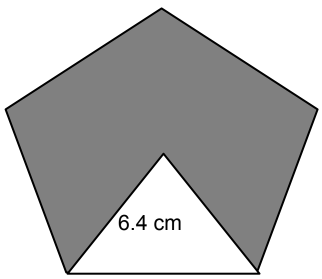
5. Alexis purchased a plot of land in England on which to graze cattle and horses that was in the shape of a regular pentagon. The land office gave him a diagram of the plot below. Calculate the perimeter of the plot in **feet** to determine the amount of fencing Alexis must purchase. Calculate the area of the plot in **square miles**. (3.28 feet = 1 meter, 1 mile = 1.61 kilometers)

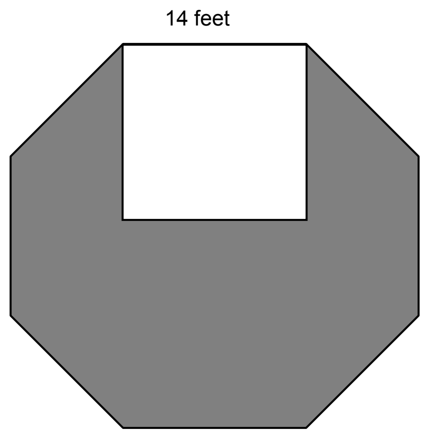


Perimeter = 65,993.6 feet
Area = 10.75 square miles

Applications of Regular Polygons (pp. 1 of 2)

1. Find the area of the shaded regions. Assume that all polygons are regular. Round to the nearest tenth.





2. Mary is tiling the entry to her new house in white tiles. In the center of the white tiles, she wants to place blue tiles in the form of a regular hexagon. The perimeter of the hexagon will be 216 inches. How many square **meters** of area will the hexagon occupy? (1 inch = 2.54 centimeter, 1 meter = 100 centimeters)

Applications of Regular Polygons (pp. 2 of 2)

3. Bill is building a garden in the shape of a regular octagon that has an area of 482.8 square yards. If the apothem measures approximately 12 yards, what is the perimeter of the garden? How many **feet** of fencing will Bill need to enclose the garden?

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   a. If the radius of the circle is 10 feet, what would be the surface area of the pool itself?

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   c. Assuming the pool's dimensions are increased one and one-half times the original size, by what factor will the area increase?

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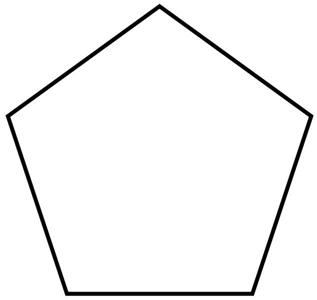
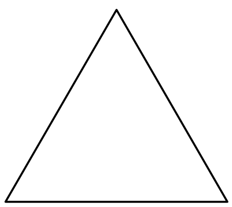
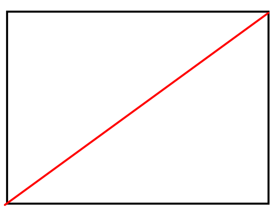
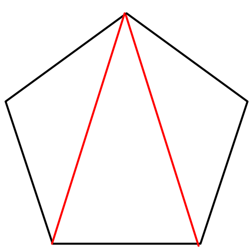
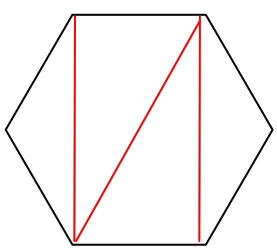
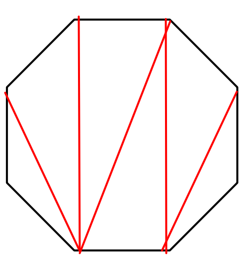


4024 meters

Pulling Together Polygons (pp. 1 of 3) KEY

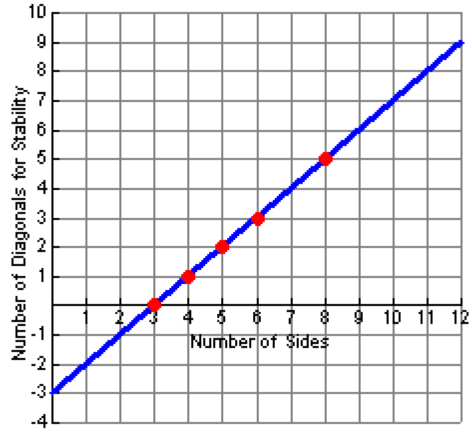
Triangles are the simplest polygons and are also some of the most useful polygons. Triangles are used for creating stability in construction. Larger polygons are broken down into triangles to increase their stability. Study the figures below and determine the least number of diagonals that must be drawn to break the figure up into parts that are only triangular, making it a stable structure.

Figure Diagonals can be drawn in multiple ways. Samples given.	**Name of Figure**	**Number of Sides**	**Least Number of Diagonals to Make the Figure Stable**
	Triangle	3	0 (already stable because it is a triangle)
	Rectangle	4	1
	Pentagon	5	2
	Hexagon	6	3
	Octagon	8	5

Pulling Together Polygons (pp. 2 of 3) KEY

1. Plot a scatterplot of the least number of diagonals needed to make the figure stable and the number of sides in the figure.

# of Sides	# of Diagonals for Stability
3	0
4	1
5	2
6	3
8	5



2. Determine a function model that can represent the situation. Plot the function on the scatterplot.
   $y = x - 3$

3. Use the function to determine the least number of diagonals needed to make a 36-sided figure stable.
   $y = (36) - 3$
   $y = 33$
   In order to make a 36-sided figure stable you would need at least 33 diagonals.

4. Would a figure ever require 25 diagonal strips to make it stable? If so, how many sides would that figure contain assuming 25 was the least number of diagonals needed to make the figure stable?
   $25 = x - 3$
   $28 = x$
   A 28-sides figure would require 25 diagonal strips to make it stable.

The Pentagon in Washington D.C., home to the Department of Defense, is one of the most recognizable buildings to people living in the United States because of its unique five sided shape. The Pentagon is actually an arrangement of five concentric pentagonal rings that form the buildings which house offices. Assume that the Pentagon is a regular polygon, and each outer wall has a length of 921 ft.

5. Find the interior angle sum of the Pentagon.
   540°

6. Find the measure of one of its interior angles.
   108°

Pulling Together Polygons (pp. 3 of 3) KEY

7. What is the exterior angle sum of the Pentagon?
   360°

8. Find the measure of one of its exterior angles.
   72°

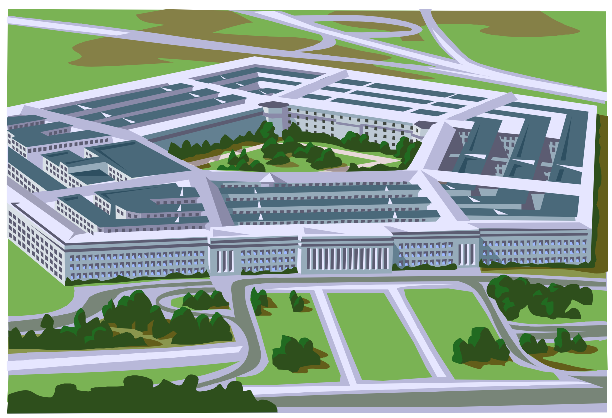
9. Find the perimeter of the outer wall.
   4605 ft.

10. Find the area contained within the perimeter of the outer wall of the Pentagon.
    1,459,379 $ft.^2$

11. Calculate the perimeter in meters and the area in square meters. (3.28 feet ≈ 1 meter)
    Perimeter ≈ 1404 meters
    Area ≈ 135,650 square meters

The concentric pentagonal rings that form the Pentagon are labeled from the interior outward as Ring A, Ring B, Ring C, Ring D and Ring E. In problem 1 you found the perimeter of the outside of Ring E and the area contained inside the outer wall of Ring E. Suppose the length of the inside of the inner most wall, that of Ring A, is $\frac{2}{5}$ that of the outermost wall of the Pentagon.

12. What is the perimeter of the innermost wall?
    1842 ft.

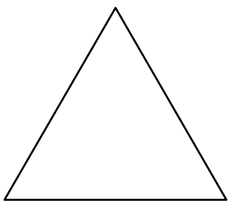
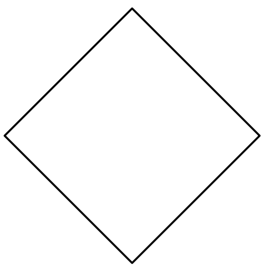
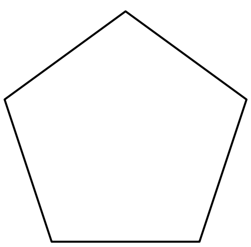
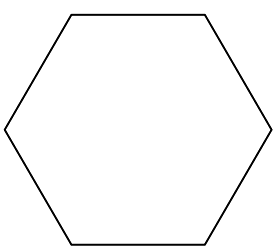
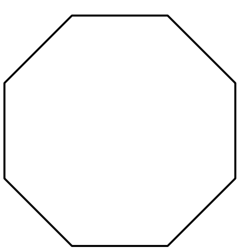


13. What is the area contained inside the innermost wall of the Pentagon or Ring A?
    233,501 $ft.^2$

14. What is the ratio of areas of the two pentagons? Explain the significance of the ratios of the areas in terms of the scale factor $\frac{2}{5}$.
    The ratio of the areas (smaller to larger) is 233,501:1,459,379 or approximately 4:25. The ratio 4:25 is the square of the ratio 2:5. Changing a figure's dimensions by some scale factor *x* results in an area change of x^2. (Students have studied effects of dimensional change in middle school. They may or may not make this connection.)

Pulling Together Polygons (pp. 1 of 3)

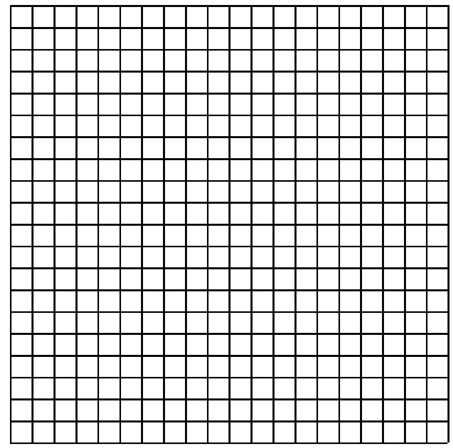
Triangles are the simplest polygons and are also some of the most useful polygons. Triangles are used for creating stability in construction. Larger polygons are broken down into triangles to increase their stability. Study the figures below and determine the least number of diagonals that must be drawn to break the figure up into parts that are only triangular, making it a stable structure.

Figure	Name of Figure	Number of Sides	Least Number of Diagonals to Make the Figure Stable
			
			
			
			
			

Pulling Together Polygons (pp. 2 of 3)

1. Plot a scatterplot of the least number of diagonals needed to make the figure stable and the number of sides in the figure.

# of Sides	# of Diagonals for Stability



2. Determine a function model that can represent the situation. Plot the function on the scatterplot.

3. Use the function to determine the least number of diagonals needed to make a 36-sided figure stable.

4. Would a figure ever require 25 diagonal strips to make it stable? If so, how many sides would that figure contain assuming 25 was the least number of diagonals needed to make the figure stable?

The Pentagon in Washington D.C., home to the Department of Defense, is one of the most recognizable buildings to people living in the United States because of its unique five sided shape. The Pentagon is actually an arrangement of five concentric pentagonal rings that form the buildings which house offices. Assume that the Pentagon is a regular polygon, and each outer wall has a length of 921 ft.

5. Find the interior angle sum of the Pentagon.

6. Find the measure of one of its interior angles.

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7. What is the exterior angle sum of the Pentagon?

8. Find the measure of one of its exterior angles.

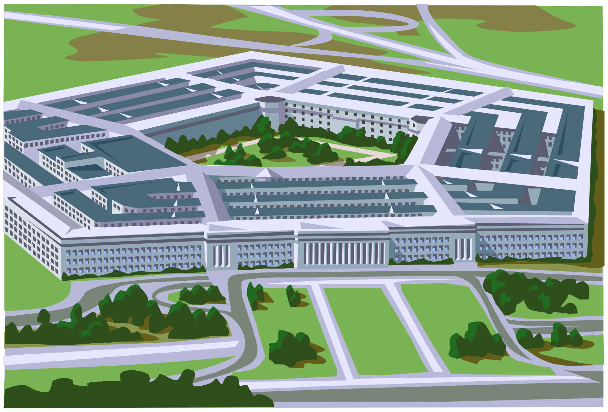
9. Find the perimeter of the outer wall in meters.

10. Find the area contained within the perimeter of the outer wall of the Pentagon.

11. Calculate the perimeter in meters and the area in square meters. (3.28 feet ≈ 1 meter)

The concentric pentagonal rings that form the Pentagon are labeled from the interior outward as Ring A, Ring B, Ring C, Ring D and Ring E. In problem 1 you found the perimeter of the outside of Ring E and the area contained inside the outer wall of Ring E. Suppose the length of the inside of the inner most wall, that of Ring A, is $\frac{2}{5}$ that of the outermost wall of the Pentagon.

12. What is the perimeter of the innermost wall?



13. What is the area contained inside the innermost wall of the Pentagon or Ring A?

14. What is the ratio of areas of the two pentagons? Explain the significance of the ratios of the areas in terms of the scale factor $\frac{2}{5}$.